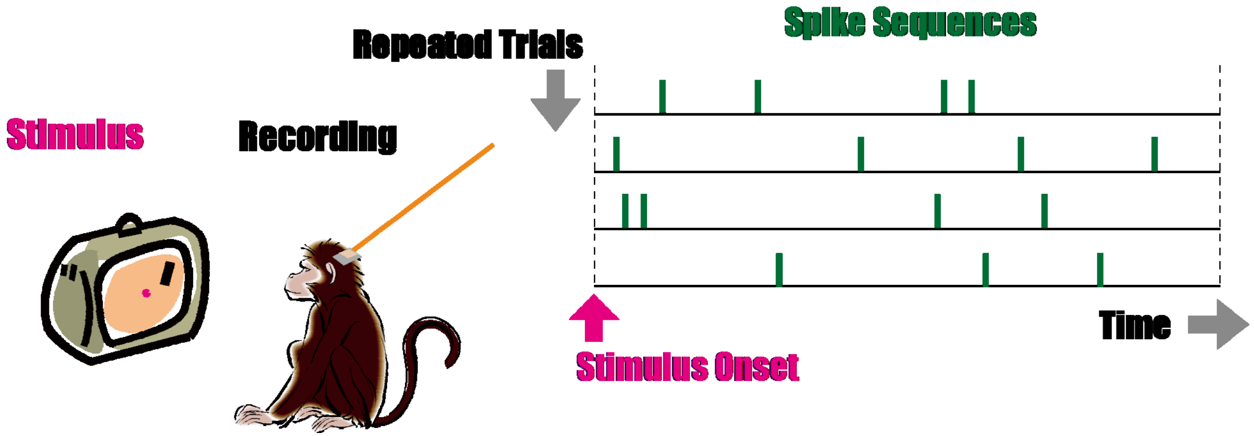
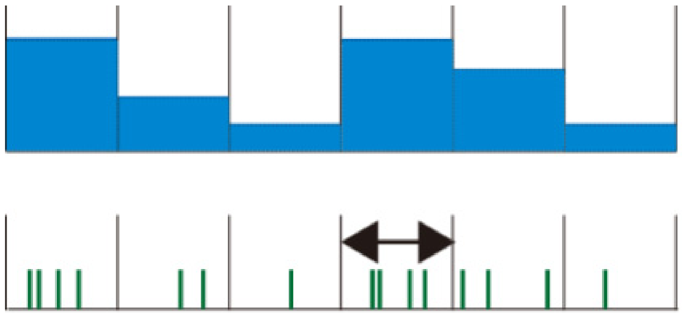
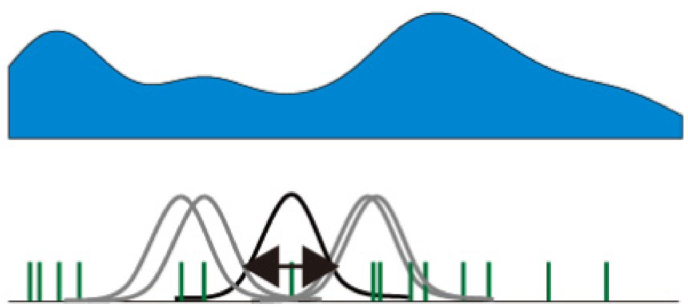
Spike-density Estimation



Histogram Method



Kernel Method



Binwidth optimization

Shimazaki & Shinomoto
Neural Computation, 2007

Bandwidth optimization

Shimazaki & Shinomoto
J. Comput. Neurosci. 2010

Abstract

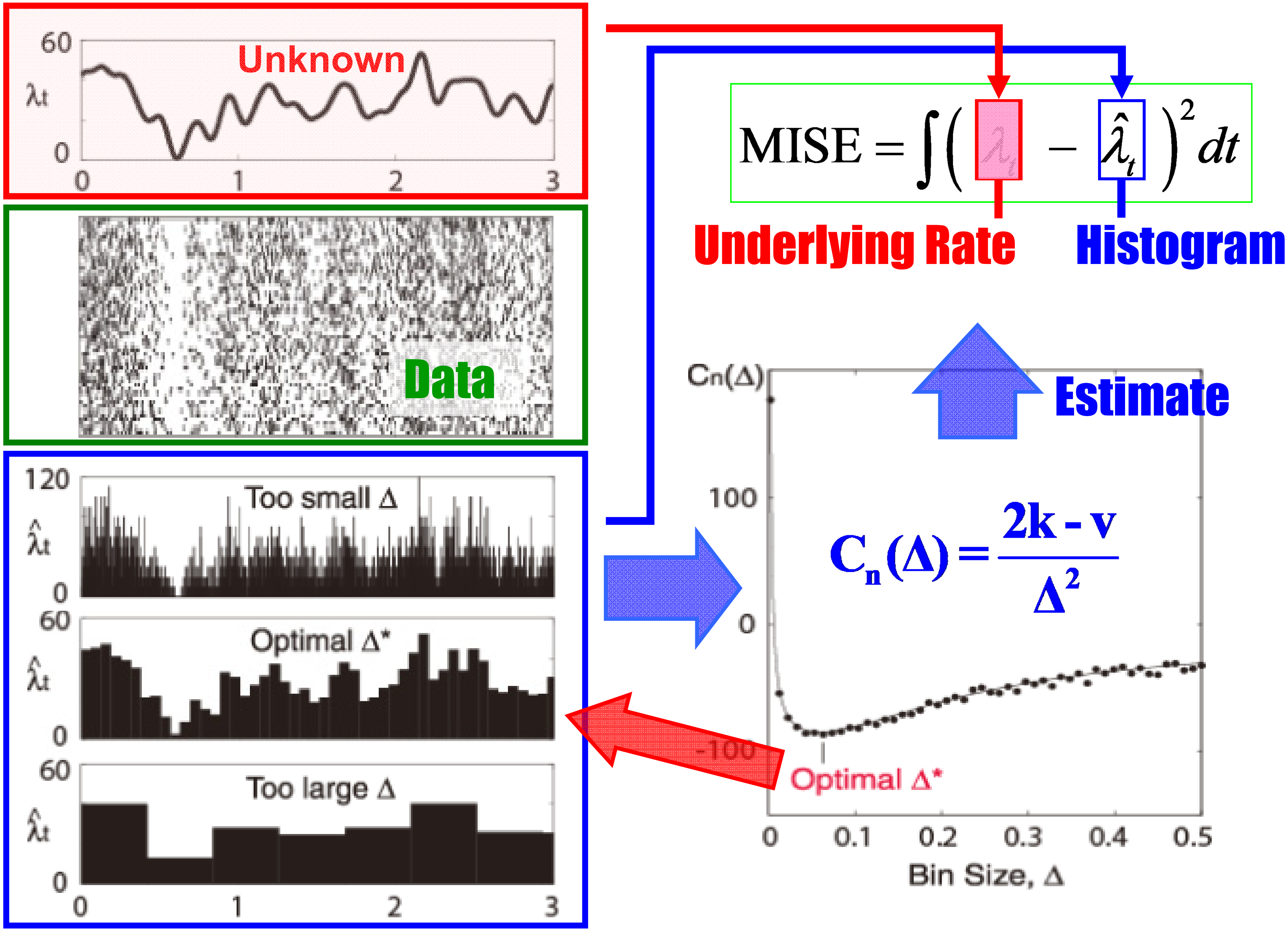
Histogram and **kernel method** have been used as standard tools for capturing the instantaneous rate of neuronal spike discharges in Neurophysiological community.

These methods are left with one free parameter that determines the smoothness of the estimated rate, namely a **binwidth** or a **bandwidth**.

In most of the neurophysiological literature, however, the binwidth or the bandwidth that critically determines **the goodness of the fit** of the estimated rate to the underlying rate has been selected by individual researchers in an unsystematic manner.

Here we established a method for **optimizing** the histogram binwidth as well as the kernel bandwidth, with which the estimated rate best approximates the unknown underlying rate.

Histogram Optimization



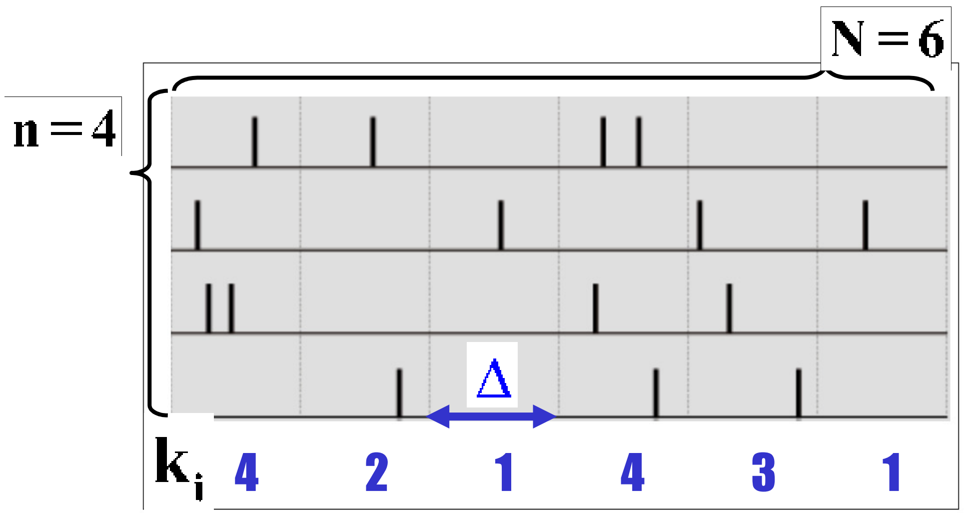
Method for Selecting the Bin Size

- Divide the data range into N bins of width Δ. Count the number of events k_i in the i th bin.
- Compute the cost function

$$C_n(\Delta) = \frac{2k - v}{(n\Delta)^2},$$

while changing the bin size Δ.

- Find Δ^* that minimize the cost function.

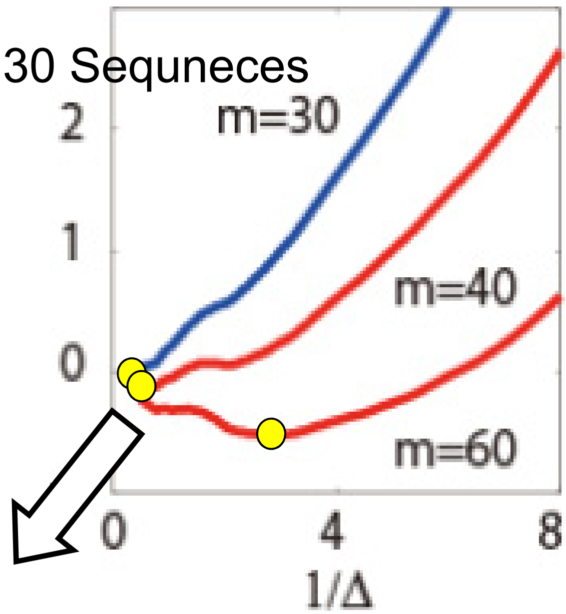


Mean $\quad k = \frac{1}{N}\sum_{i=1}^{N} k_i,$

Variance $\quad v = \frac{1}{N}\sum_{i=1}^{N} (k_i - k)^2$

Shimazaki & Shinomoto Neural Computation, 2007

Extrapolation Method



Original: $C_n(\Delta)$

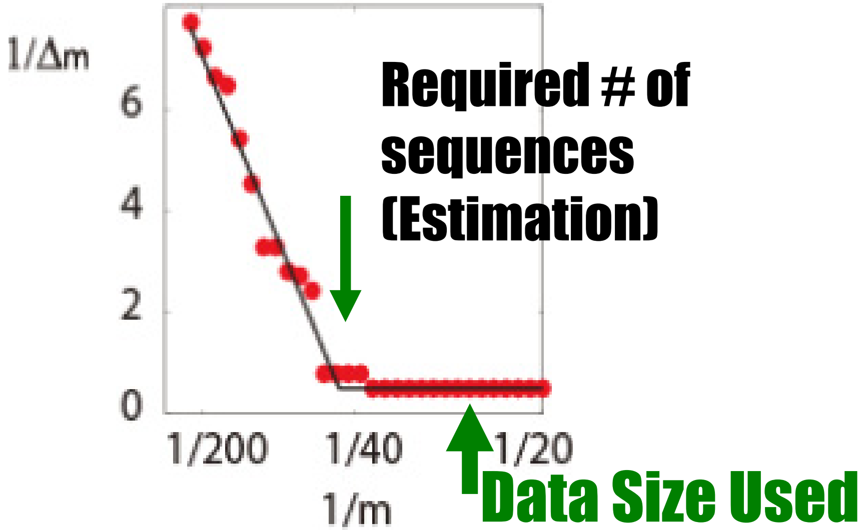
Optimal bin size diverges

Extrapolated:

$$C_m(\Delta\,|\,n) = \left(\frac{1}{m} - \frac{1}{n}\right)\frac{k}{n\Delta^2} + C_n(\Delta)$$

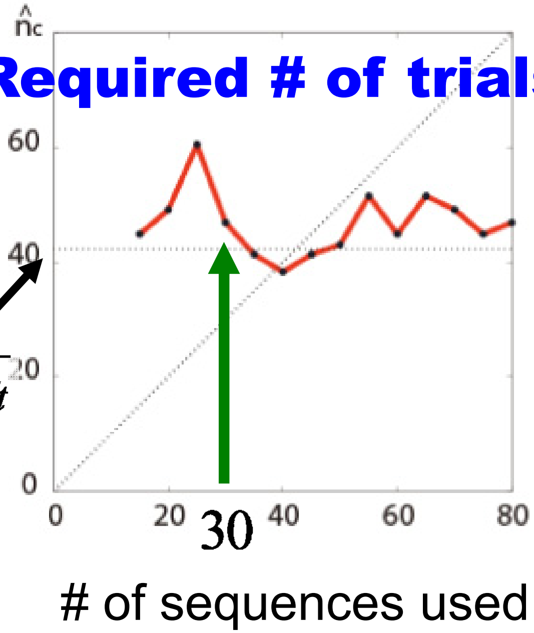
Finite optimal bin size

Optimal bin size v.s. m

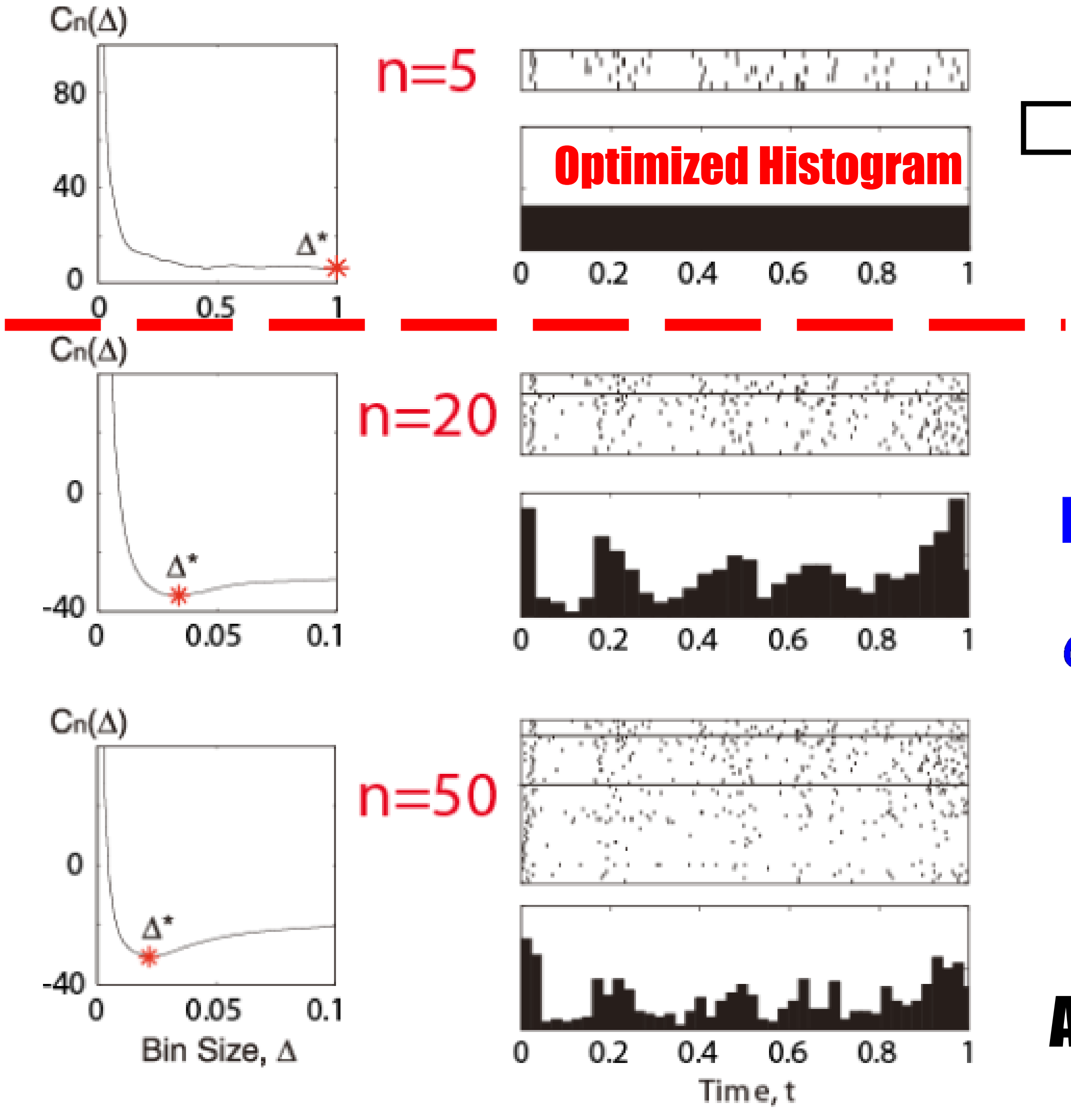


Required # of trials

$$n_c = \frac{\mu}{\int_{-\infty}^{\infty} \phi(t)\,dt}$$



Extrapolation Method



Too few to make a Histogram !

Extrapolation

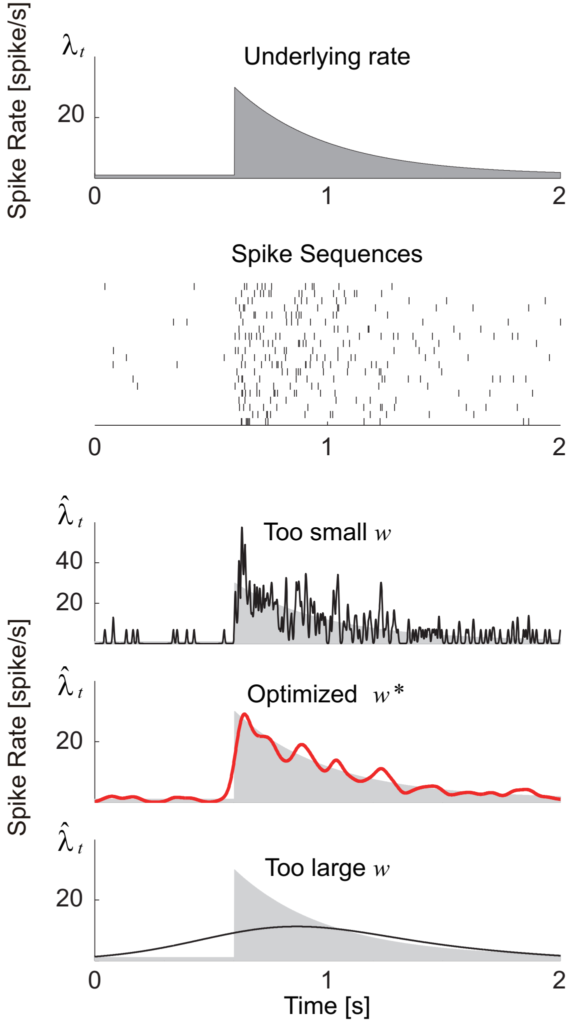
$$C_m(\Delta \mid n) = \left(\frac{1}{m} - \frac{1}{n} \right) \frac{k}{n\Delta^2} + C_n(\Delta)$$

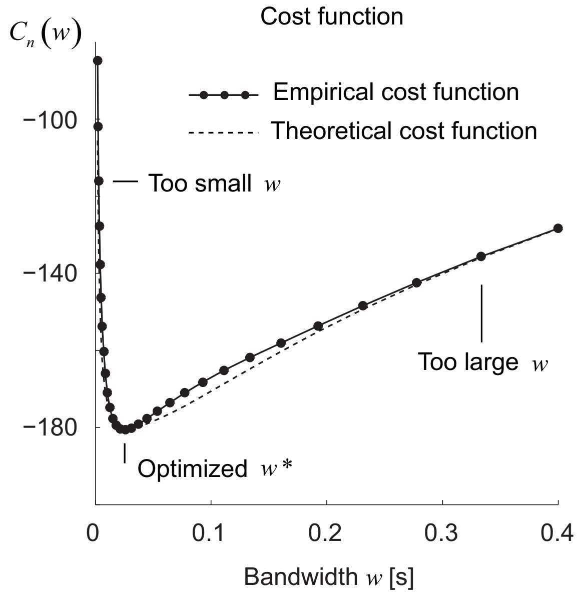
Estimation:

At least 12 trials are required.

Data : Britten et al. (2004) neural signal archive

Kernel Optimization





$$C(w) = \sum_{i,j} \int k_w(s-t_i) k_w(s-t_j) ds - 2 \sum_{i \neq j} k_w(t_i, t_j)$$

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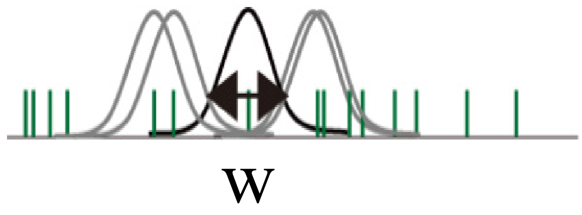
Kernel Optimization Formula

For a Gauss kernel function,

$$k_w(x) = \frac{1}{\sqrt{2\pi}w}\exp\left(-\frac{x^2}{2w^2}\right)$$



find w that minimizes the formula,

Total number of spikes

$$C(w) = \frac{N}{w} - \frac{1}{w}\sum_{i<j}\left\{4\sqrt{2}e^{-(t_i-t_j)^2/2w^2} - 2e^{-(t_i-t_j)^2/4w^2}\right\}.$$

Bandwidth

Spike Time

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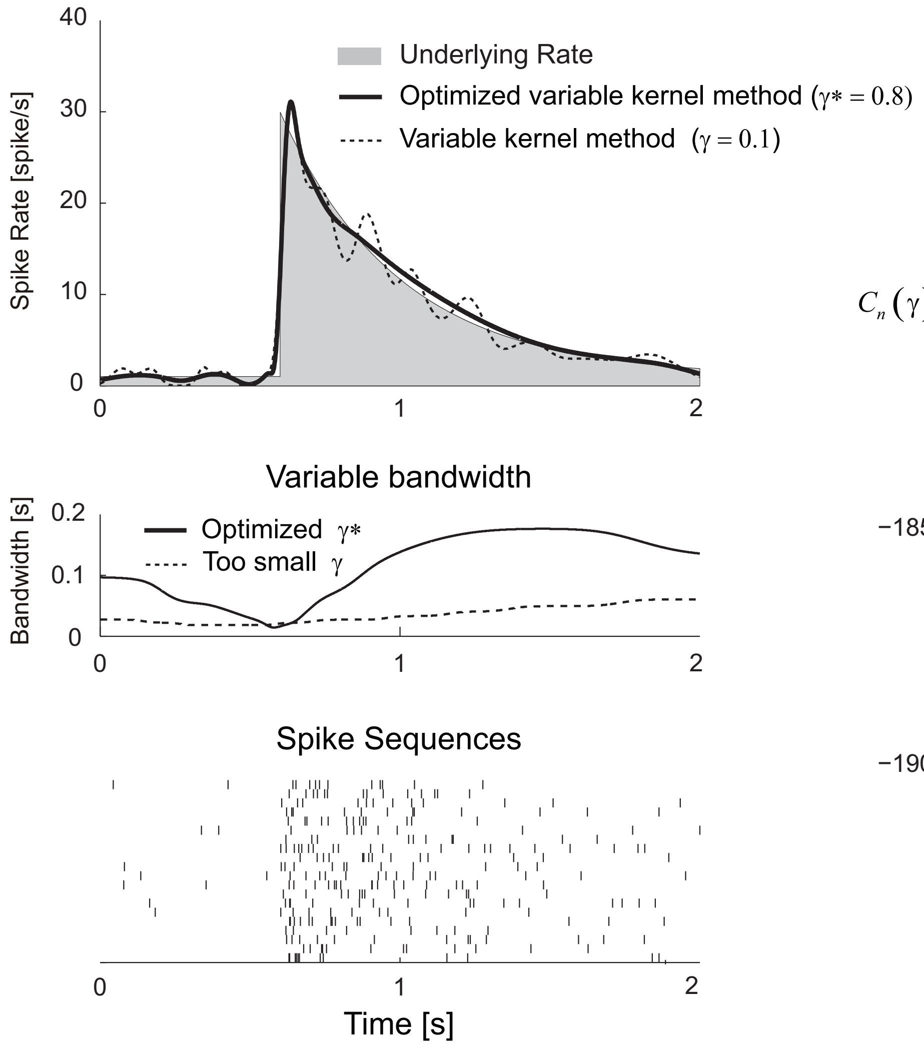
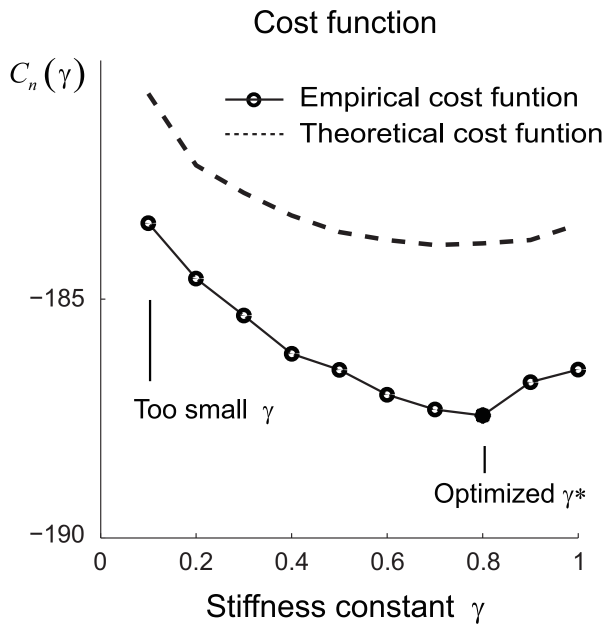
Variable Kernel Method





Our method automatically adjusts the stiffness of bandwidth variability

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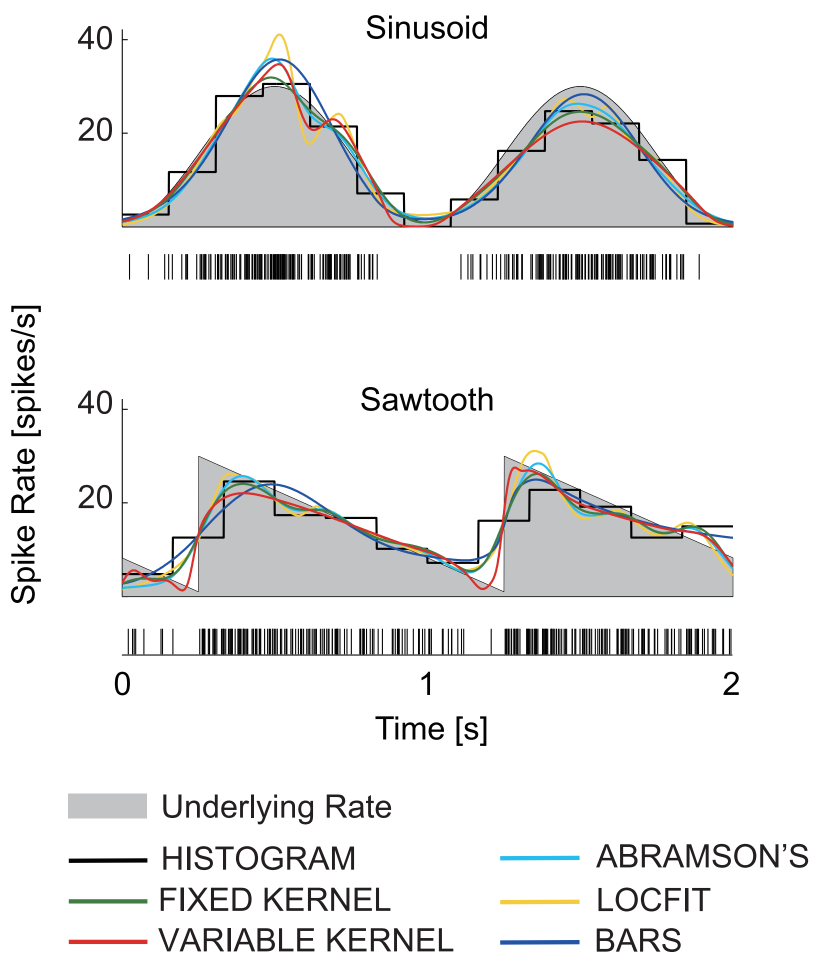
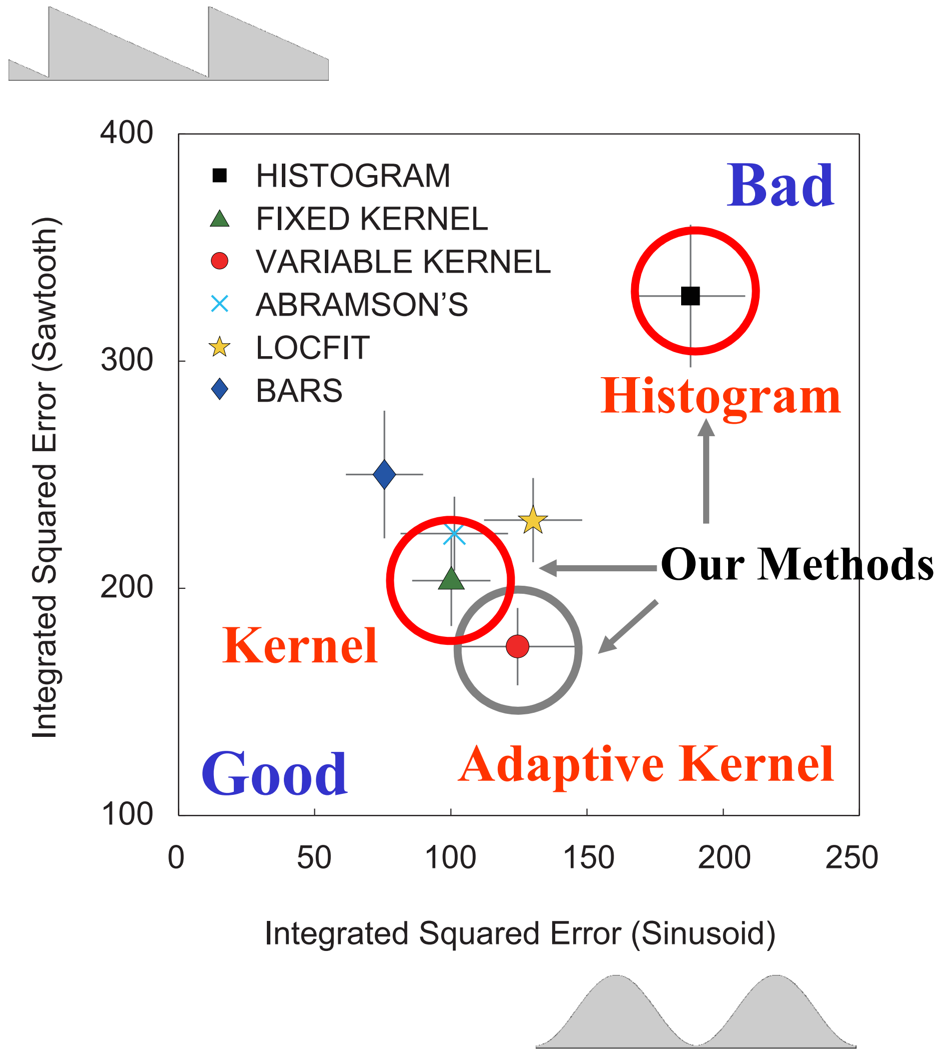
Performance Comparison





Kernel method is comparable to, or even better than modern methods.

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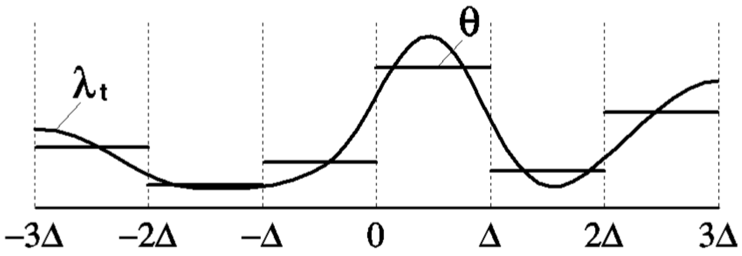
Appendix

Histogram construction

Time-Varying Rate



The mean underlying rate in an interval [0, Δ]:

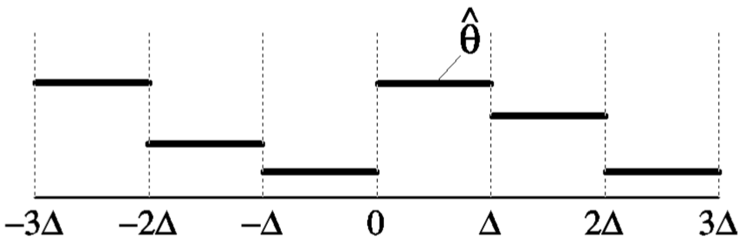
$$\theta = \frac{1}{\Delta}\int_0^{\Delta} \lambda_t \, dt.$$

Spike Sequences

The spike count in the bin obeys the Poisson distribution*:

$$p(k \mid n\Delta\theta) = \frac{(n\Delta\theta)^k}{k!} e^{-n\Delta\theta}.$$

Time Histogram



A histogram bar-height is an estimator of θ:

$$\hat{\theta}_n = \frac{k}{n\Delta}$$

*When the spikes are obtained by repeating an independent trial, the accumulated data obeys the Poisson point process due to a general limit theorem.

Theory for Histogram Optimization

$$\mathrm{MISE} \equiv \frac{1}{T}\int_0^T E(\hat{\lambda}_t - \lambda_t)^2 dt = \left\langle \frac{1}{\Delta}\int_0^\Delta E(\underbrace{\hat{\theta}_n - \lambda_t}_{\theta - \theta})^2 dt \right\rangle.$$

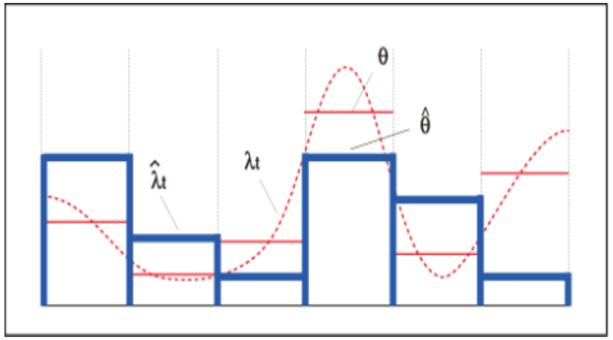
Expectation by the Poisson statistics, given the rate.

Average over segmented bins.

$$\mathrm{MISE} = \left\langle E(\hat{\theta}_n - \theta)^2 \right\rangle + \frac{1}{\Delta}\int_0^\Delta \left\langle (\lambda_t - \theta)^2 \right\rangle dt.$$

Sampling Error

Systematic Error



Decomposition of the Systematic Error

$$\frac{1}{\Delta}\int_0^\Delta \left\langle (\overbrace{\lambda_t - \theta}^{\langle\theta\rangle - \langle\theta\rangle})^2 \right\rangle dt = \frac{1}{\Delta}\int_0^\Delta \left\langle (\lambda_t - \langle\theta\rangle)^2 \right\rangle dt - \left\langle (\theta - \langle\theta\rangle)^2 \right\rangle$$

Systematic Error

Variance of the rate

Independent of Δ

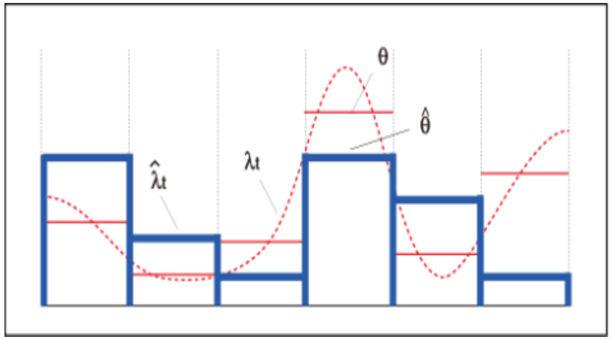
Variance of
an ideal histogram

Introduction of the cost function:

$$C_n(\Delta) \equiv \mathrm{MISE} - \frac{1}{T}\int_0^T \left(\lambda_t - \langle\theta\rangle\right)^2 dt$$

$$= \left\langle E(\hat{\theta}_n - \theta)^2 \right\rangle - \left\langle \left(\theta - \langle\theta\rangle\right)^2 \right\rangle.$$

Sampling error Unknown: Variance of an ideal histogram



The variance decomposition: $\left\langle E\left(\hat{\theta}_n - \langle E\hat{\theta}_n\rangle\right)^2 \right\rangle = \left\langle E(\hat{\theta}_n - \theta)^2 \right\rangle + \left\langle \left(\theta - \langle\theta\rangle\right)^2 \right\rangle.$

Variance of a histogram Sampling error

$$C_n(\Delta) = 2\left\langle E\left(\hat{\theta}_n - \theta\right)^2 \right\rangle - \left\langle E\left(\hat{\theta}_n - \langle E\hat{\theta}_n\rangle\right)^2 \right\rangle.$$

The Poisson statistics obeys: $E(\hat{\theta}_n - \theta)^2 = \frac{1}{n\Delta} E\hat{\theta}_n.$

$$C_n(\Delta) = \frac{2}{n\Delta}\left\langle E\hat{\theta}_n \right\rangle - \left\langle E\left(\hat{\theta}_n - \langle E\hat{\theta}_n\rangle\right)^2 \right\rangle.$$

Mean of a Histogram Variance of a Histogram